Math 55 Discussion problems 2 May

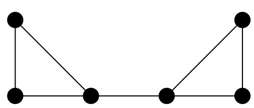
1. Show that in every simple graph there is a path from every vertex of odd degree to some other vertex of odd degree.

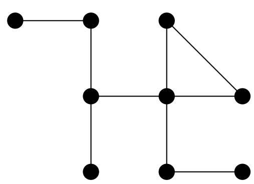
2. Show that if a connected simple graph G is the union of the graphs G_1 and G_2, then G_1 and G_2 have at least one common vertex.

3. The distance between two distinct vertices v_1 and v_2 of a connected simple graph is the length (number of edges) of the shortest path between v_1 and v_2. The radius of a graph is the minimum over all vertices v of the maximum distance from v to another vertex. The diameter of a graph is the maximum distance between two distinct vertices. Find the radius and diameter of

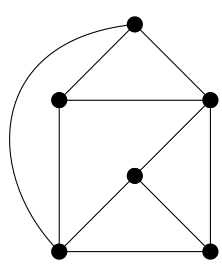
   (a) K_6 (b) $K_{4,5}$ (c) Q_3 (d) C_6

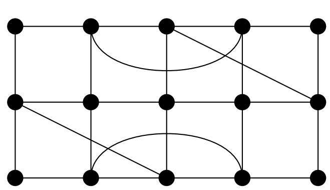
4. Find all the cut vertices and cut edges of the given graphs

   (a) 

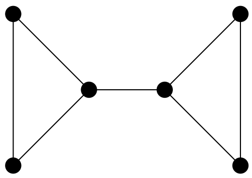
   (b) 

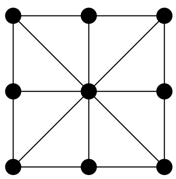
5. Determine whether each of the given graphs have an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

   (a) 

   (b) 

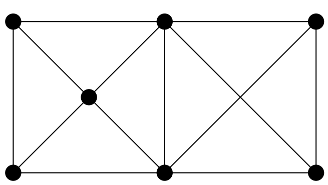
6. Determine whether each of the given graphs has a Hamilton circuit. If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists.

(a) 

(b) 

7. A knight is a chess piece that can move either two spaces horizontally and one space vertically or one space horizontally and two spaces vertically. That is, a knight on square (x, y) can move to any of the eight squares $(x \pm 2, y \pm 1), (x \pm 1, y \pm 2)$, if these squares are on the chessboard. A knight's tour is a sequence of legal moves by a knight starting at some square and visiting each square exactly once.
We can model knight's tours using the graph that has a vertex for each square on the board, with an edge connecting two vertices if a knight can legally move between the squares represented by these vertices. Draw the graph that represents the legal moves of a knight on a 3×3 chessboard. Show that there is no knight's tour on a 3×3 chessboard.

8. A clique in a simple undirected graph is a complete subgraph that is not contained in any larger complete subgraph. Find all cliques in the graph shown.

(a) 

(b)